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Evaluation of Kowalski's Method of Calculating Stresses at Internal Thread Reliefs

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This paper examines the accuracy and limitation of Kowalski's method of calculating stresses at thread reliefs by correlating its results against finite element data over a certain range of thread geometries. Curves defining the range of validity of the method in terms of thread sizes and engagement lengths are presented. The effect of assuming different stress concentration factors is also investigated. Difficulties and limitations associated with the finite element approach are briefly discussed. The study is based on the MIL-S-008879B 12UNJ class 3B internal threads.

Nomenclature

A = area
c = see Eq. (7)
D = diameter
E = elastic modulus
e = base of natural logarithm
f = stress
h = moment arm
K_t = stress concentration factor
L = thread engagement length
M = bending moment
P = thread load
R = radius
r = fillet radius
t = thickness at thread relief section
α = see Eq. (7)
β = see Eq. (8)
ν = Poisson ratio

Subscripts

a = axial
b = bending
m = mean
o = outer
p = pitch
r = thread relief
T = total

Introduction

PRIOR to the advent of the finite element technique of structural analysis, stresses at the thread relief of many mechanical parts were calculated using Kowalski's method.[1] Even with the increasing popularity of computers and finite element (FE) programs, the method, mainly because of its simplicity in application, continues to be widely used in practice. This paper attempts to evaluate the accuracy, limitation, and validity of Kowalski's method by correlating its results against data generated by the COSMOS/M FE software. In view of the large number of thread series that are used in industry, it would be quite impossible to perform the correlation across the entire range of thread sizes and geometries that are encountered in various applications. The present study is focused specifically on the MIL-S-008879B class 3B series.[2] In the unified inch thread series, there are three classes of internal threads. All have clearance fits that enable the threads to be assembled without interference. Class 3B threads have the highest precision and provide the tightest fit. This is a commonly adopted series in the design of many mechanical and hydraulic components used in the aircraft industry. Thread major diameters ranging from 0.875 in. (22.225 mm) to 4.00 in. (101.6 mm) based on the internal 12 thread series per 12UNJ class 3B are investigated.

Although SI conversions for the basic analytical parameters are included for completeness, it must be emphasized that the present results are applicable only to the aforementioned thread series. Equivalent metric series and sizes, even if they exist, generally contain some differences in geometry that may render the present results invalid.

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Review of Kowalski's Method

Kowalski's method is briefly reviewed to highlight the important assumptions involved in the derivation of the governing equations. A typical internal thread configuration is shown in Fig. 1. The relevant symbols are defined:

D_m = mean diameter at thread relief section *A-A*
D_p = pitch diameter
D_r = inside diameter at thread relief section *A-A*
D_o = outside diameter at thread relief section *A-A*
h = distance to midsection of thread relief measured from thread pitch diameter
P = thread load

The total stress f_T across section *A-A* comprises an axial component f_a and a bending component f_b all multiplied by a factor K_t to account for the effect of stress concentration as shown in Eq. (1)

$$f_T = K_t(f_a + f_b) \tag{1}$$

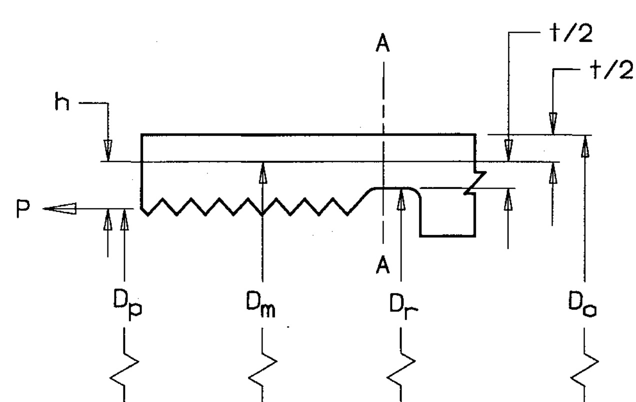

Fig. 1 Definition of thread geometry.

where f_a and f_b are given by

$$f_a = P/A_r \tag{2}$$

$$f_b = 6M\alpha/t^2 \tag{3}$$

M is the meridional bending moment per unit circumference, t the thickness at the thread relief, and A_r the cross-sectional area at the thread relief. These are given by

$$M = Ph/(\pi D_m) \tag{4}$$

$$t = (D_o - D_r)/2 \tag{5}$$

$$A_r = \pi(D_o^2 - D_r^2)/4 \tag{6}$$

The quantity α in Eq. (3) is derived by analogy to a beam of semi-infinite length supported on an elastic foundation and given by

$$\alpha = (0.25/c)[1 + 4e^{-c}\sin c - e^{-2c} - e^{-2c}(\sin 2c + \cos 2c)] \tag{7}$$

where

$$c = \beta L$$

$$\beta = [12(1-\nu^2)/(D^2t^2)]^{0.25} \tag{8}$$

L = thread engagement length

ν = Poisson ratio

K_t in Eq. (1) is dependent on the type of thread adjacent to the thread relief since the highest stress concentration occurs where the last thread joins the thread relief. Values assumed in practice range from 3 to 4.

Finite Element Correlation

A general-purpose FE software, COSMOS/M Ver 1.65, is used to provide data for correlation with Kowalski's method. The geometry of the finite element model (FEM) is shown in Fig. 2. Relevant symbols are defined as:

R_p = thread pitch radius
R_r = inside radius at thread relief section

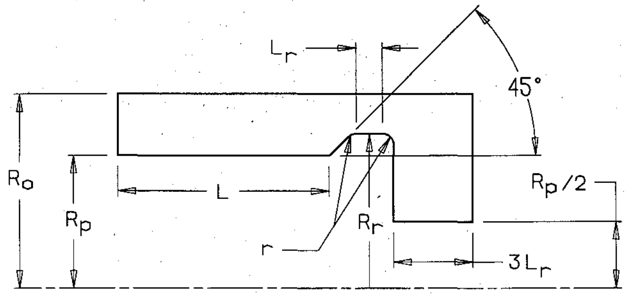

Fig. 2 FEM geometry.

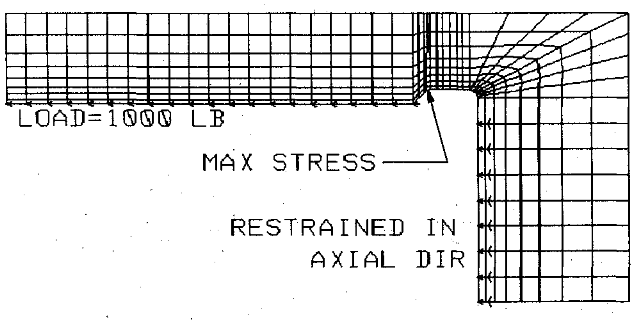

Fig. 3 Details of FE mesh used.

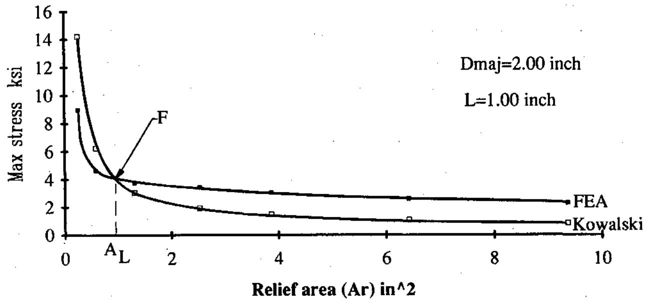

Fig. 4 Correlation between FE analyses and Kowalski's method with a K_t of 3.0.

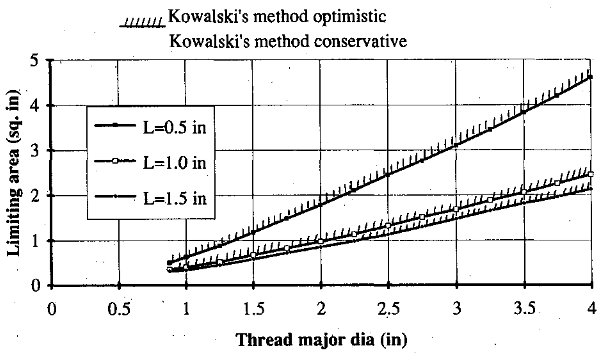

Fig. 5 Validity of Kowalski's method for K_t of 3.0.

R_o = outside radius at thread relief section
L_r = length of thread relief
r = fillet radius

Dimensions used to construct the thread relief represent typical configurations encountered in practice and are generally dictated by manufacturing requirements. The value of L_r is taken to be 1.5 times thread pitch. The fillet radii r at the corners of the relief are 0.02 in. (0.508 mm), and R_r is 0.01 in. (0.254 mm) greater than the thread major radius.

The FE mesh together with load and boundary conditions are shown in Fig. 3.

The FEM mesh utilized consists of 423 nodes and 368 quadrilateral axisymmetric elements. A load of 1000 lb (4448.22 N) distributed equally to the nodes representing the entire thread engagement length at the thread pitch radius is applied to all analyses. The reason for adopting this simplistic load distribution is discussed later. Material plasticity is not included in the analyses.

Four thread sizes with major diameters of 0.875 in. (22.225 mm), 2.00 in. (50.8 mm), 3.00 in. (76.2 mm), and 4.00 in. (101.6 mm) are considered. The reason for choosing the 0.875- over the 1.00-in. thread is because the 1.00-in. thread is not classified as a basic primary thread. For each thread size, engagement lengths of 0.5 in. (12.7 mm), 1.0 in. (25.4 mm), and 1.5 in. (38.1 mm) are studied. For each combination of thread size and thread engagement, the effect of varying the value of R_o on the thread relief stress is investigated.

Material constants used in the FE analysis are based on steel with a value of 29E6 psi (199.95 kN/mm^2) for E and 0.3 for ν. Stresses obtained from linear FE solutions are insensitive to changes in E, but do vary slightly with changes to ν. However, for most aerospace alloys such as those of aluminum and titanium, ν is relatively constant and does not deviate significantly from 0.3.

The location of the maximum axial stress obtained from the FE analysis is also shown in Fig. 3. Fatigue cracks encountered in practical designs are generally initiated where the last thread joins the relief. Although the actual thread form is not included in the FE model, the location of maximum stress is nevertheless quite realistic.

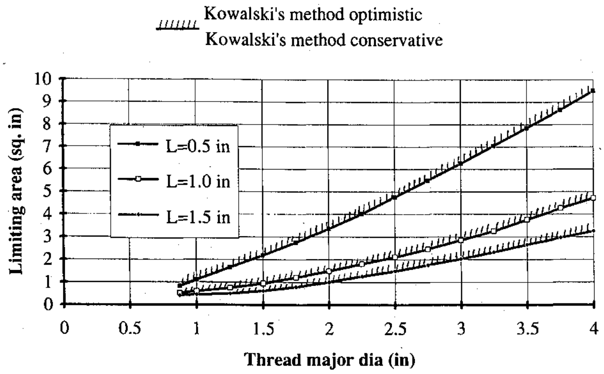

Fig. 6 Validity of Kowalski's method for K_t of 4.0.

Discussion

A total of 84 different geometries are investigated using both Kowalski's method and FE analyses. Results from Kowalski's method based on a K_t of 3 and from FE analyses are shown in Fig. 4 for a configuration with a thread major diameter of 2.0 in. (50.8 mm) and thread engagement length of 1.0 in. (25.4 mm). The figure illustrates the variation of calculated axial stresses as the thread relief area A_r is increased.

It may be seen that for small values of A_r, Kowalski's method is conservative for design purposes since it overpredicts the stress at the thread relief. As A_r is increased, Kowalski's method becomes less conservative and at point F it agrees exactly with the FE result. The thread relief area corresponding to point F is denoted by A_L and represents a limiting value above which Kowalski's method may actually produce optimistic and hence unsafe designs by underpredicting the maximum thread relief stress.

Repeating the foregoing exercise for different values of thread major diameters and engagement lengths establishes a range of values for A_L and defines an envelope of thread geometries for which Kowalski's method may be safely applied. Values of A_L obtained by assuming a K_t of 3 are shown in Fig. 5.

If a higher K_t is assumed, it has the effect of shifting Kowalski's curve in Fig. 4 upward and increasing the value of A_L. This increases the conservatism of the method for small values of A_r, but increases the range of thread geometries for which the method may be safely applied. Values for A_L obtained by assuming a K_t of 4 are shown in Fig. 6.

In practice, the highest K_t occurs where the last thread joins the thread relief. Neither Kowalski's method nor the present FE analyses include the effect of the actual thread form. The accurate evaluation of the exact K_t is a very difficult exercise and not much relevant data are listed in textbooks and design charts. It is reasonable to expect that even for parts manufactured with identical nominal dimensions, actual stress concentration will vary due to the inevitable effect of machine tolerances. For these reasons, the rational selection of K_t relies more on prior experience than any other factor. For some static load applications, a K_t of 1 may be used, provided the material selected for the design has sufficient ductility to undergo plastic yielding to "blunt out" the notch effect of the geometric discontinuity. However, in aircraft applications, static failures do not generally occur in thread reliefs since they are often preceded by fatigue crack propagation resulting from repeated operational loads.

For most ductile alloys under low-cycle-fatigue situations, the present linear FE analyses may inherently overpredict the stresses at the thread relief since the assumption of linearity cannot accommodate the almost inevitable development of localized yielding as discussed by the present author.[3] However, since the actual thread form is omitted in the FE analysis, the real stress concentration effect may not be fully simulated and calculated results may underpredict actual maximum stresses in certain circumstances. Fortunately, these two effects tend to cancel each other and the author has successfully utilized the present FE approach in the design of many internal thread reliefs machined from various materials including steel, aluminum, and titanium.

A further limitation of the FE model may be attributed to the assumption that the applied load is uniformly distributed over the entire thread engagement length. The exact load distribution along the thread engagement is complex and a function of the relative stiffness of and lubrication between the mating parts, as well as thread dimensional tolerances. It is reasonable to expect the threads nearest the thread relief to carry the major portion of the loading. Some designers even assume the entire load to be taken by the first four or five threads. A simplistic uniform distribution is, however, chosen for the present correlation since this is also the distribution assumed by Kowalski's method. The accuracy and validity of this assumption can only be verified with experimental testing.

From an analytical standpoint, it is obviously desirable to include in the FE model the actual thread form and effect of material nonlinearity to simulate the real K_t. The mating external threads would have to be included in the model and interface gap elements used to transfer the thread loading. This more rigorous approach, even though it cannot address the uncertainties regarding thread dimensional tolerances, will simulate more accurately the effect of stress concentration and load distribution among threads. Such an approach will be adopted as part of a future numerical investigation to study thread load distribution and to quantify the magnitude of stress concentration at thread reliefs.

It is important to remember that the data presented in this paper are applicable only to the MIL-S-008879B 12UNJ class 3B internal series with the specified thread relief configuration. The results may not be valid for other thread series or thread relief geometries, including nominally equivalent metric series.

Conclusions

The accuracy and validity of Kowalski's method of analyzing stresses at thread reliefs are examined in the present paper for a certain range of thread geometries and configurations by correlation with results from an FE program. For small thread relief areas A_r, the method is found to provide conservative results, whereas for a larger value of A_r, the stresses obtained may actually lead to unsafe designs. The limiting value of A_r for which the method may be safely applied is established for a range of thread sizes. Increasing the value of the assumed stress concentration has the effect of making the method applicable to a wider range of geometries, but the drawback is greater conservatism for smaller values of A_r. The limitation associated with the simplistic linear FE solution is also discussed. The present study is based on the MIL-S-008879B 12UNJ class 3B internal threads. Extreme caution should be exercised if the present results are to be extrapolated to other thread series since changes in thread forms or thread relief configurations will have a significant effect on the magnitude and distribution of stresses.

References

[1]Kowalski, S.,"Strength of Threaded Undercuts," *Design News*, Nov. 1957.

[2]Anon., "General Specification for Screw Threads, Controlled Radius Root with Increased Minor Diameter," (USAF) MIL-S-008879B, July 1988.

[3]Tsang, S. K.,"Making Sense of FEA," *Machine Design*, Vol. 65, No. 6, 1993, pp. 76–80.